Simple Linear Regression

Inference

Prof. Maria Tackett

STA 210

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Topics

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- Conduct a hypothesis test for β_1

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- Conduct a hypothesis test for β_1

- Calculate a confidence interval for β_1

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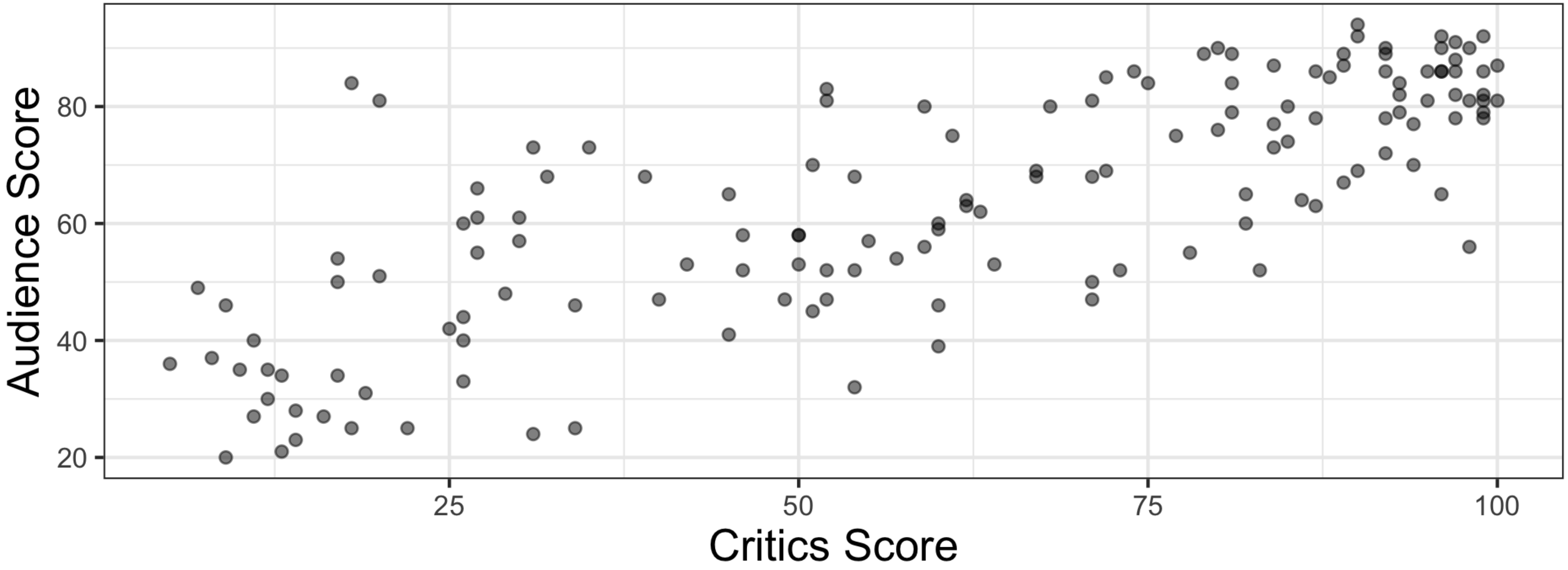
Movie ratings data

The data set contains the "Tomatometer" score (**`critics`**) and audience score (**`audience`**) for 146 movies rated on rottentomatoes.com.



The model

```
model <- lm(audience ~ critics, data = movie_scores)
```

```
model %>%
  tidy() %>%
  kable(format = "html", digits = 3)
```

term	estimate	std.error	statistic	p.value
(Intercept)	32.316	2.343	13.795	0
critics	0.519	0.035	15.028	0

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The model

$$\widehat{\text{audience}} = 32.316 + 0.519 \times \text{critics}$$

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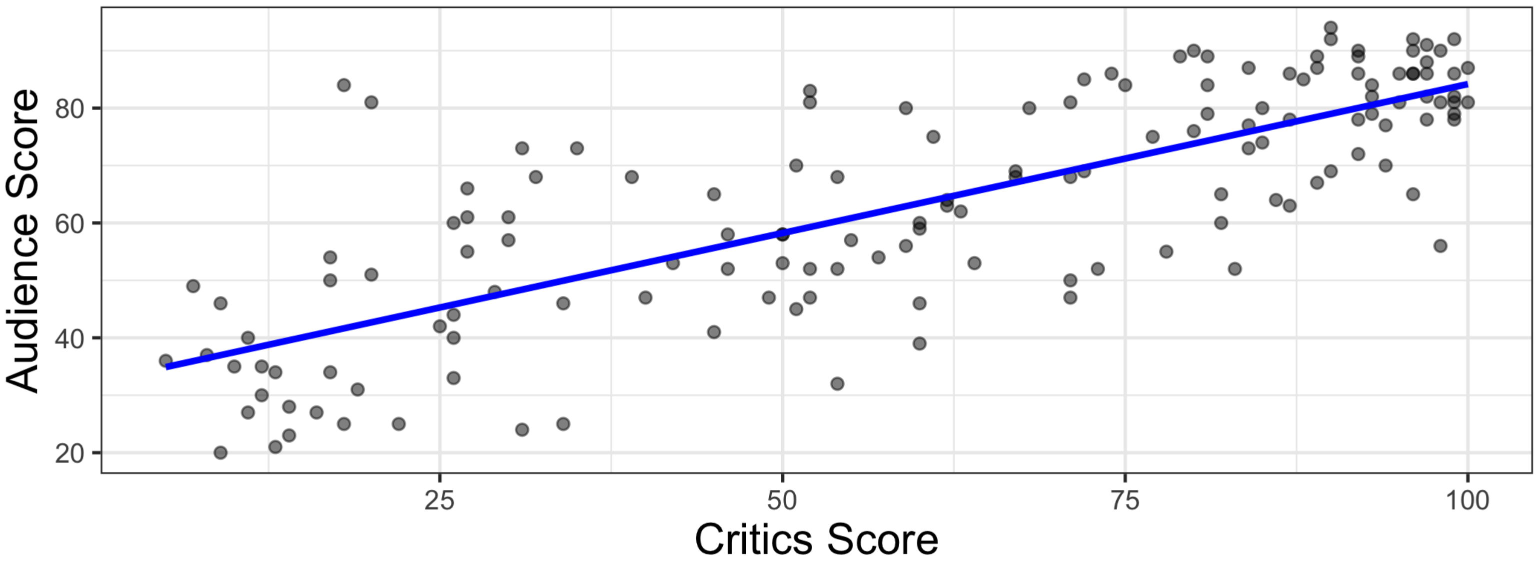

Does the data provide sufficient evidence that β_1 is significantly different from 0?

Outline of a hypothesis test

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1️⃣ State the hypotheses.

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2️⃣ Calculate the test statistic.

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4️⃣ State the conclusion.

1️⃣ State the hypotheses

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$$H_0 : \beta_1 = 0$$
$$H_a : \beta_1 \neq 0$$

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1️⃣ State the hypotheses

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Null hypothesis

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1️⃣ State the hypotheses

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(Intercept)	32.316	2.343	13.795	0
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$$H_0 : \beta_1 = 0$$
$$H_a : \beta_1 \neq 0$$

Null hypothesis

Alternative hypothesis

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2️⃣ Calculate the test statistic

term	estimate	std.error	statistic	p.value
(Intercept)	32.316	2.343	13.795	0
critics	0.519	0.035	15.028	0

$$\text{test statistic} = \frac{\text{Estimate} - \text{Hypothesized}}{\text{Standard error}}$$

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2️⃣ Calculate the test statistic

term	estimate	std.error	statistic	p.value
(Intercept)	32.316	2.343	13.795	0
critics	0.519	0.035	15.028	0

$$t = \frac{\hat{\beta}_1 - 0}{SE_{\hat{\beta}_1}}$$

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2️⃣ Calculate the test statistic

term	estimate	std.error	statistic	p.value
(Intercept)	32.316	2.343	13.795	0
critics	0.519	0.035	15.028	0

$$t = \frac{\hat{\beta}_1 - 0}{SE_{\hat{\beta}_1}}$$

$$t = \frac{0.5187 - 0}{0.0345} = \mathbf{15.03}$$

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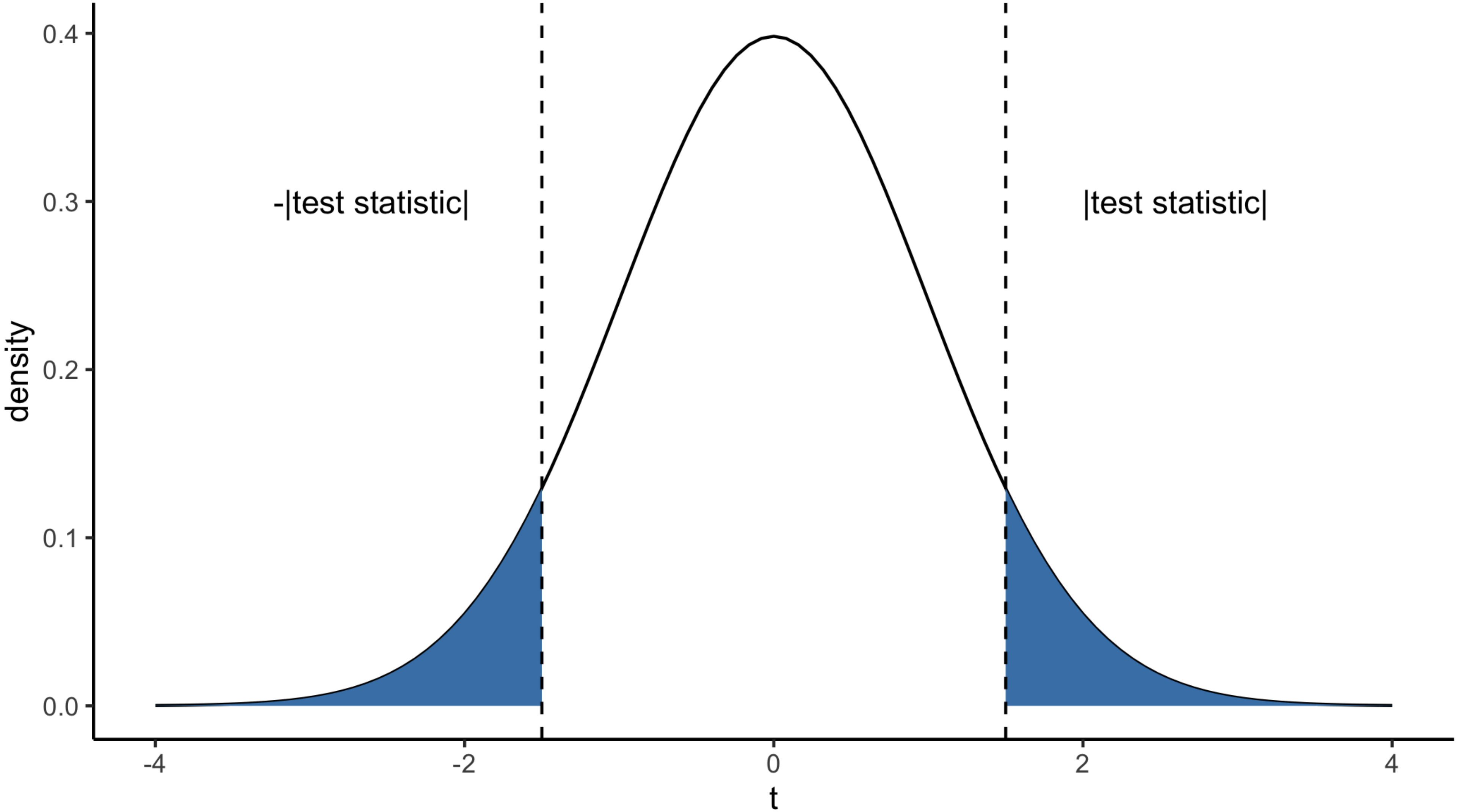
3️⃣ Calculate the p-value

term	estimate	std.error	statistic	p.value
(Intercept)	32.316	2.343	13.795	0
critics	0.519	0.035	15.028	0

$$\text{p-value} = P(|t| \geq |\text{test statistic}|)$$

Calculated from a t distribution with $n - 2$ degrees of freedom

3️⃣ Calculate the p-value



Understanding the p-value

Magnitude of p-value	Interpretation
p-value < 0.01	strong evidence against H_0
0.01 < p-value < 0.05	moderate evidence against H_0
0.05 < p-value < 0.1	weak evidence against H_0
p-value > 0.1	effectively no evidence against H_0

These are general guidelines. The strength of evidence depends on the context of the problem.

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4️⃣ State the conclusion

term	estimate	std.error	statistic	p.value
(Intercept)	32.316	2.343	13.795	0
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4️⃣ State the conclusion

term	estimate	std.error	statistic	p.value
(Intercept)	32.316	2.343	13.795	0
critics	0.519	0.035	15.028	0

The data provide sufficient evidence that the population slope β_1 is different from 0.

There is a linear relationship between the critics score and audience score for movies on rottentomatoes.com.

What is a plausible range of values for the population slope β_1?

Confidence interval for β_1

$$\text{Estimate} \pm \ (\text{critical value}) \ \times \text{SE}$$

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Confidence interval for β_1

$$\text{Estimate} \pm \text{(critical value)} \times \text{SE}$$

$$\hat{\beta}_1 \pm t^* \times SE_{\hat{\beta}_1}$$

t^* is calculated from a t distribution with $n-2$ degrees of freedom

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Calculating the 95% CI for β_1

term	estimate	std.error	statistic	p.value
(Intercept)	32.316	2.343	13.795	0
critics	0.519	0.035	15.028	0

$$\hat{\beta}_1 = 0.519 \qquad t^* = 1.977 \qquad SE_{\hat{\beta}_1} = 0.035$$

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Calculating the 95% CI for β_1

term	estimate	std.error	statistic	p.value
(Intercept)	32.316	2.343	13.795	0
critics	0.519	0.035	15.028	0

$$\hat{\beta}_1 = 0.519 \qquad t^* = 1.977 \qquad SE_{\hat{\beta}_1} = 0.035$$

$$0.519 \pm 1.977 \times 0.035$$

$$\mathbf{[0.450, 0.588]}$$

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Interpretation

$$[0.450, 0.588]$$

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Interpretation

$$[0.450, 0.588]$$

We are 95% confident that for every one point increase in the critics score, the audience score is predicted to increase on average between 0.450 and 0.588 points.

Recap

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- Conducted a hypothesis test for β_1

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