Multiple linear regression

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STA 210

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Topics

- Introduce multiple linear regression
- Interpret a coefficient $\hat{\beta}_j$
- Use the model to calculate predicted values and the corresponding interval

House prices in Levittown

The data set contains the sales price and characteristics of 85 homes in Levittown, NY that sold between June 2010 and May 2011.

Levittown was built right after WWII and was the first planned suburban community built using mass production techniques.

The article "Levittown, the prototypical American suburb – a history of cities in 50 buildings, day 25" gives an overview of Levittown's controversial history.

Analysis goals

We would like to use the characteristics of a house to understand variability in the sales price.

To do so, we will fit a **multiple linear regression model**

Using our model, we can answers questions such as

- What is the relationship between the characteristics of a house in Levittown and its sale price?
- Given its characteristics, what is the expected sale price of a house in Levittown?

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The data

```
## # A tibble: 10 x 7
##    bedrooms bathrooms living_area lot_size year_built property_tax sale_price
##       <dbl>     <dbl>       <dbl>    <dbl>      <dbl>        <dbl>      <dbl>
##  1        4       1          1380     6000       1948         8360     350000
##  2        4       2          1761     7400       1951         5754     360000
##  3        4       2          1564     6000       1948         8982     350000
##  4        5       2          2904     9898       1949        11664     375000
##  5        5       2.5        1942     7788       1948         8120     370000
##  6        4       2          1830     6000       1948         8197     335000
##  7        4       1          1585     6000       1948         6223     295000
##  8        4       1           941     6800       1951         2448     250000
##  9        4       1.5        1481     6000       1948         9087     299990
## 10        3       2          1630     5998       1948         9430     375000
```

Variables

Predictors

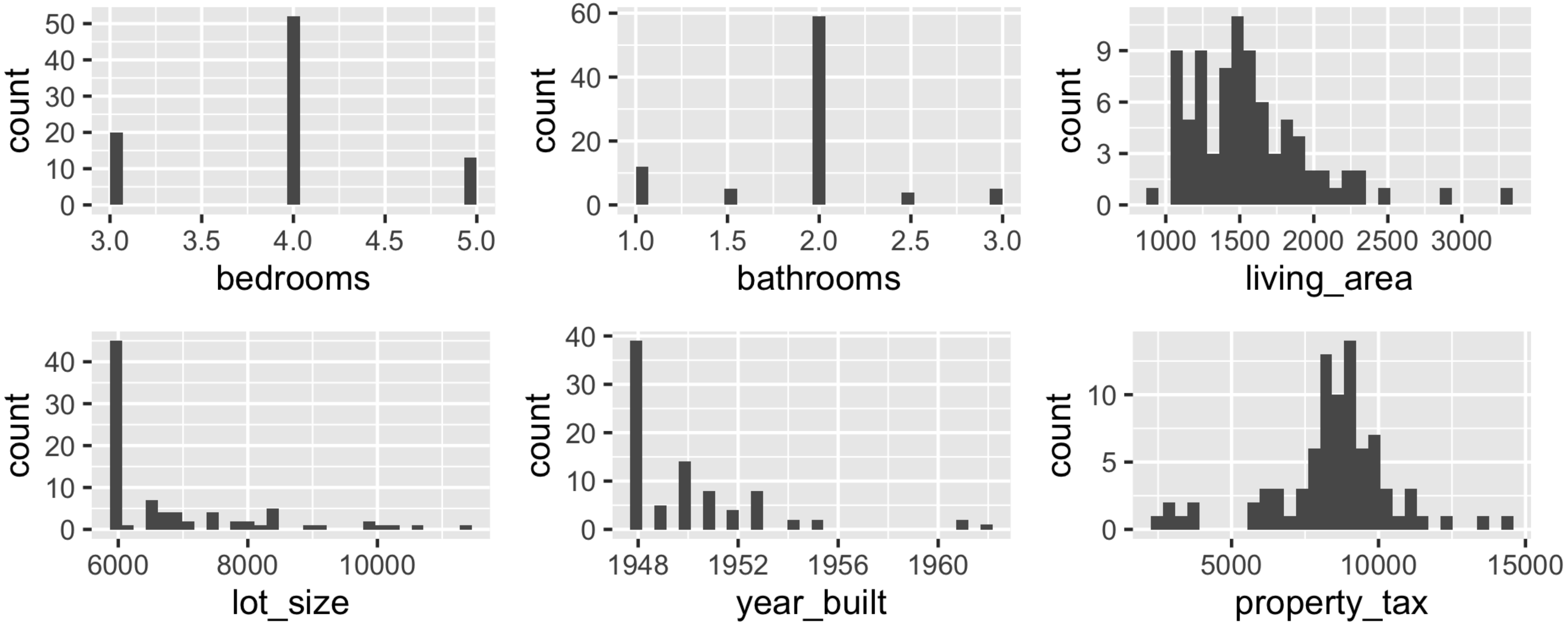
- `bedrooms`: Number of bedrooms
- `bathrooms`: Number of bathrooms
- `living_area`: Total living area of the house (in square feet)
- `lot_size`: Total area of the lot (in square feet)
- `year_built`: Year the house was built
- `property_tax`: Annual property taxes (in U.S. dollars)

Response

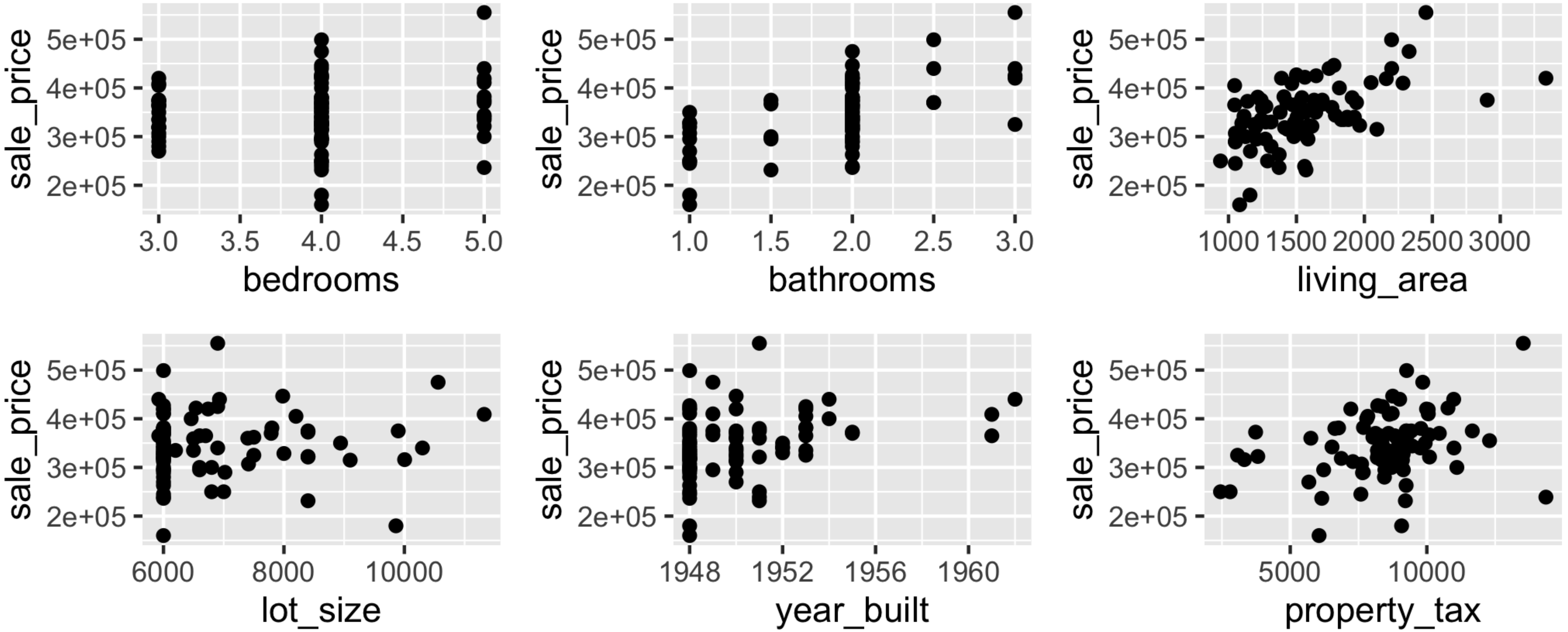
- `sale_price`: Sales price (in U.S. dollars)

EDA: Response variable

EDA: Predictor variables



EDA: Response vs. Predictors



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So far we've used a *single predictor variable* to understand variation in a quantitative response variable

Now we want to use *multiple predictor variables* to understand variation in a quantitative response variable

Multiple linear regression (MLR)

Based on the analysis goals, we will use a **multiple linear regression** model of the following form

$$
\begin{aligned}
\widehat{\text{sale\_price}} &= \hat{\beta}_0 + \hat{\beta}_1 \text{bedrooms} + \hat{\beta}_2 \text{bathrooms} + \hat{\beta}_3 \text{living\_area} \\
&\quad + \hat{\beta}_4 \text{lot\_size} + \hat{\beta}_5 \text{year\_built} + \hat{\beta}_6 \text{property\_tax}
\end{aligned}
$$

Similar to simple linear regression, this model assumes that at each combination of the predictor variables, the values `sale_price` follow a Normal distribution

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Regression Model

- **Recall:** The simple linear regression model assumes

$$Y|X \sim N(\beta_0 + \beta_1 X, \sigma_\epsilon^2)$$

- **Similarly:** The multiple linear regression model assumes

$$Y|X_1, X_2, \ldots, X_p \sim N(\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p, \sigma_\epsilon^2)$$

For a given observation $(x_{i1}, x_{i2} \dots, x_{ip}, y_i)$

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip} + \epsilon_i \quad \epsilon_i \sim N(0, \sigma_\epsilon^2)$$

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Regression Model

- At any combination of the predictors, the mean value of the response Y, is

$$\mu_{Y|X_1,\ldots,X_p} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p$$

- Using multiple linear regression, we can estimate the mean response for any combination of predictors

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 + \cdots + \hat{\beta}_p X_p$$

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Home price model

term	estimate	std.error	statistic	p.value
(Intercept)	-7148818.957	3820093.694	-1.871	0.065
bedrooms	-12291.011	9346.727	-1.315	0.192
bathrooms	51699.236	13094.170	3.948	0.000
living_area	65.903	15.979	4.124	0.000
lot_size	-0.897	4.194	-0.214	0.831
year_built	3760.898	1962.504	1.916	0.059
property_tax	1.476	2.832	0.521	0.604

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$$
\begin{aligned}
\widehat{\text{price}} = & -7148818.957 - 12291.011 \times \text{bedrooms} \\
& + 51699.236 \times \text{bathrooms} + 65.903 \times \text{living area} \\
& - 0.897 \times \text{lot size} + 3760.898 \times \text{year built} \\
& + 1.476 \times \text{property tax}
\end{aligned}
$$

Interpreting $\hat{\beta}_j$

- The estimated coefficient $\hat{\beta}_j$ is the expected change in the mean of y when x_j increases by one unit, ***holding the values of all other predictor variables constant***.

- **Example:** The estimated coefficient for `living_area` is 65.90. This means for each additional square foot of living area, we expect the sale price of a house in Levittown, NY to increase by $65.90, on average, holding all other predictor variables constant.

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Prediction

Example: What is the predicted sale price for a house in Levittown, NY with 3 bedrooms, 1 bathroom, 1050 square feet of living area, 6000 square foot lot size, built in 1948 with $6306 in property taxes?

```
-7148818.957 - 12291.011 * 3 + 51699.236 * 1 +
  65.903 * 1050 - 0.897 * 6000 + 3760.898 * 1948 +
  1.476 * 6306
```

```
## [1] 265360.4
```

The predicted sale price for a house in Levittown, NY with 3 bedrooms, 1 bathroom, 1050 square feet of living area, 6000 square foot lot size, built in 1948 with $6306 in property taxes is **$265,360**.

Intervals for predictions

Just like with simple linear regression, we can use the `predict` function in R to calculate the appropriate intervals for our predicted values

```
x0 <- data.frame(bedrooms = 3, bathrooms = 1,
                 living_area = 1050, lot_size = 6000,
                 year_built = 1948,
                 property_tax = 6306)
```

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Confidence interval for $\hat{\mu}_y$

Calculate a 95% confidence interval for the **estimated mean price** of houses in Levittown, NY with 3 bedrooms, 1 bathroom, 1050 square feet of living area, 6000 square foot lot size, built in 1948 with $6306 in property taxes:

```
predict(price_model, x0, interval = "confidence",
        level = 0.95)
```

```
##          fit      lwr      upr
## 1 265360.2 238481.7 292238.7
```

Prediction interval for $\hat{y}$

Calculate a 95% prediction interval for an individual house in Levittown, NY with 3 bedrooms, 1 bathroom, 1050 square feet of living area, 6000 square foot lot size, built in 1948 with $6306 in property taxes:

```
predict(price_model, x0, interval = "prediction",
        level = 0.95)
```

```
##        fit      lwr      upr
## 1 265360.2 167276.8 363443.6
```

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🛑 Cautions

- **Do not extrapolate!** Because there are multiple predictor variables, there is the potential to extrapolate in many directions
- The multiple regression model only shows **association, not causality**
  - To show causality, you must have a carefully designed experiment or carefully account for confounding variables in an observational study

Recap

- Introduced multiple linear regression
- Interpreted a coefficient $\hat{\beta}_j$
- Used the model to calculate predicted values and the corresponding interval